OFDM Narrowband Interference Estimation Using Cyclic Prefix Based Algorithm

Cormac de Fréin, Mark Flanagan, Anthony Fagan
School of Electronic Engineering,
University College Dublin, Dublin, Ireland
cdefrein@ee.ucd.ie, mark@ee.ucd.ie, tony.fagan@ucd.ie

***Abstract*— OFDM systems experience BER degradation in the presence of narrowband interference (NBI). We propose a novel NBI cancellation algorithm for a wideband OFDM receiver. Modelling the NBI as a single-tone sinusoid, subspace-based (SB) methods can be used to estimate it. However, direct application of SB methods to the received signal yields poor results except when the system is operating at very low SIR. Our algorithm exploits the redundancy inherent in the cyclic prefix of the OFDM system to remove the information bearing component of the received signal prior to the estimation of NBI by using SB methods. We estimate the NBI by applying SB methods to the resulting signal. Simulation results show a significant improvement in performance with the proposed scheme.**[1]

I. Introduction

Narrowband interference (NBI) is commonly found in communications sytems. In an OFDM system, unless the frequency of the NBI coincides with a subcarrier frequency, the spectrum of the NBI will span the whole bandwidth of the system. The presence of NBI can severly degrade the performance of the OFDM system [1]. Various algorithms have been proposed for NBI suppression. In [2], a Nyquist window function is used instead of the rectangular window commonly used in the receiver DFT. The sidelobes of the Nyquist window function are lower that those of the rectangular window. This reduces the amount of spectral spreading experienced by the NBI. In [1], OFDM is combined with frequency hopping spread spectrum. This also suppresses the NBI effectively. NBI cancellation algorithms (i.e. estimation and subtraction) are presented in [3] and [4]. Specially chosen OFDM subcarriers are left silent during transmission. The received information on these subcarriers is used to estimate the NBI.

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We propose a novel NBI cancellation algorithm for a wideband OFDM receiver. Subspace-based (SB) methods are a well established technique for estimating sinusoids corrupted by white noise [6]. Modelling the NBI as a single-tone sinusoid, SB methods can be used to estimate it. However, direct application of SB methods to the received signal yields poor results except when the system is operating at very low SIR. This is because the noise variance is difficult to estimate, particularly at high SNR, and inaccurate estimation of the noise variance leads to inaccurate estimation of the NBI parameters. Our algorithm exploits the redundancy inherent in the cyclic prefix of the OFDM system to facilitate more accurate estimation of the NBI. The information bearing component of the received signal is removed and the noise variance in the resulting signal is doubled. SB methods are then applied to the resulting signal to estimate the NBI. The larger noise variance in this signal makes it easier to estimate, which results in a more accurate estimation of the NBI parameters. Simulation results show a significant improvement in performance with the proposed scheme.

In section II the OFDM NBI system model is introduced. The deleterious effect of a single-tone NBI on OFDM reception is explained. In section III the NBI is estimated by applying SB methods directly to the received OFDM signal. A novel cyclic prefix aided SB method for NBI estimation is introduced in section IV. Finally conclusions are drawn in section V.

Notation: Throughout this paper, all vectors and matrices are denoted by **bold** characters, with vectors as lower case and matrices as upper case. Real and complex scalars are represented by normal math type.

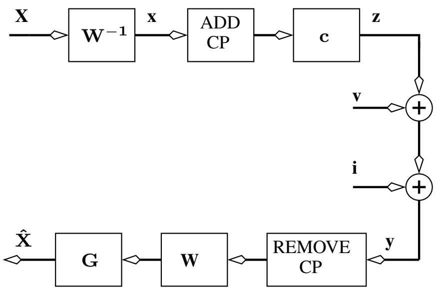

Fig. 1. Block diagram of OFDM system with NBI

II. OFDM NBI System Model

The wideband OFDM system under consideration is illustrated in figure 1. The number of subchannels is N and the cyclic prefix length is N_g. OFDM modulation and demodulation is performed by the IDFT matrix $\mathbf{W^{-1}}$ and the DFT matrix $\mathbf{W}$ respectively. The $(N \times 1)$ vector $\mathbf{X}$ consists of the complex constellation symbols, with variance σ_x^2. The Rayleigh fading channel is modelled with an L-tap FIR filter

$$\mathbf{c} = \left[\begin{array}{cccc} c_0 & c_1 & \ldots & c_{L-1} \end{array} \right]^T$$

and the channel coefficients are normalized such that

$$\sum_{i=0}^{L-1} |c_i|^2 = 1$$

The block $\mathbf{G}$ represents a frequency domain equalizer, which may be zero-forcing or MMSE. The k^{th} sample of the received signal is

$$y_k = \sum_{l=0}^{L-1} c_l x_{k-l} + i_k + v_k = z_k + i_k + v_k$$

$$\ldots \quad k = 0, 1, \ldots, N + N_g - 1$$

where i_k is the NBI, and v_k is complex AWGN of zero mean and variance σ_v^2. The samples z_k represent the information bearing component in the received signal. Assuming the bandwidth of the NBI is sufficiently narrow compared with the bandwidth of the OFDM system, it can be modelled as a single-tone sinusoid [5]

$$i_k = \alpha e^{j(\omega k + \phi)}$$

with frequency ω, phase offset ϕ and amplitude α.

If the NBI frequency coincides with a subcarrier frequency, it will interfere only with that subcarrier. However, this cannot be assumed to be true. Due to the sidelobes caused by rectangular windowing at the receiver, the spectrum of the NBI spans the bandwidth of the system and it will, in general, interfere with all subcarriers. Subcarriers in the vicinity of the NBI are more strongly affected. If the BER on these subcarriers is sufficiently high, they will dominate the BER of the overall system.

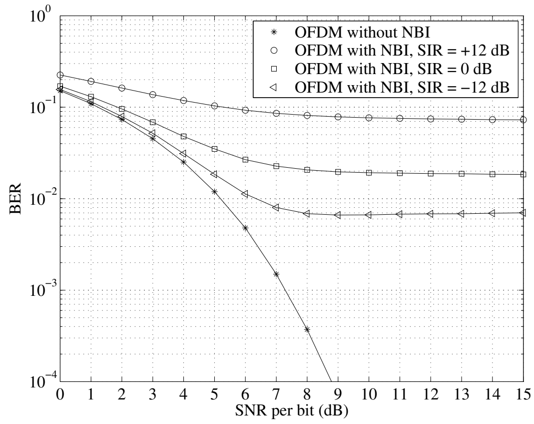

Fig. 2. BER of OFDM system with NBI of SIR's -12dB, 0dB and $+12$dB

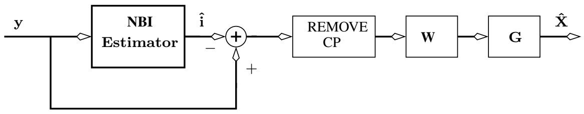

Fig. 3. OFDM receiver with NBI cancellation

Figure 2 illustrates the BER of an OFDM system corrupted by AWGN and NBI. The signal to interference ratio (SIR) is defined as

$$SIR = \frac{\sigma_x^2}{\alpha^2}$$

The data symbols are QPSK, and NBI of three different levels -12dB, 0dB and $+12$dB are shown. An error floor, which depends on the level of NBI, is observed.

In the following sections, we will describe two methods of NBI cancellation for an OFDM system. For the purposes of comparison, we assume the NBI estimation is performed on an OFDM symbol by OFDM symbol basis. Both algorithms follow the general NBI cancellation scheme shown in figure 3. Consider the received wideband OFDM signal

$$y_k = z_k + i_k + v_k = i_k + d_k$$

The NBI i_k is corrupted by a "noise" term d_k. Estimation of NBI in an OFDM signal is therefore equivalent to estimation of a sinusoid in noise. A NBI cancellation algorithm, operating before removal of the cyclic prefix, makes an estimate $\hat{i}_k$ of the NBI and subtracts this from the received signal. OFDM signal reception then proceeds as normal.

III. Narrowband Interference Cancellation by Direct Application of Subspace Based Methods

SB methods are a well established technique for estimating sinusoids in white noise [6]. It is now shown how SB methods can be used to estimate the NBI in a received OFDM signal. We denote by N_1 the number of samples used in the NBI estimation process. For per-OFDM-symbol estimation, we take $N_1 = N + N_g$.

We define $L_1 = N_1 - P + 1$ vectors, each of size $(P \times 1)$, as follows

$$\mathbf{y_k} = \begin{bmatrix} y_k & y_{k+1} & \dots & y_{k+P-1} \end{bmatrix}^T$$
$$= \alpha e^{j\phi_k}\sqrt{P}\mathbf{s} + \mathbf{d_k} \quad \dots \quad k = 0, 1, \dots, L-1$$

where

$$\mathbf{d_k} = \begin{bmatrix} d_k & d_{k+1} & \dots & d_{k+P-1} \end{bmatrix}^T$$

and

$$\mathbf{s} = \frac{1}{\sqrt{P}} \begin{bmatrix} 1 & e^{j\omega} & e^{2j\omega} & \dots & e^{(P-1)j\omega} \end{bmatrix}^T$$

The span of $\mathbf{s}$ is the signal space S and its orthogonal complement $S^{\perp}$ is the "noise" subspace.

The $(P \times P)$ autocorrelation matrix $\mathbf{C_y}$ of the received signal $\mathbf{y_k}$ is by definition

$$\mathbf{C_y} \triangleq E\left\{\mathbf{y_k y_k^H}\right\}$$
$$= \alpha^2 P \mathbf{ss^H} + \sigma_v^2 \mathbf{I} + \sigma_x^2 \mathbf{HH^H}$$

where $\mathbf{H}$ is the $(P \times (P + L - 1))$ channel matrix defined as

$$\mathbf{H} = \begin{bmatrix} c_0 & c_1 & \dots & c_{L-1} & 0 & \dots & 0 \\ 0 & c_0 & c_1 & \dots & c_{L-1} & \dots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \dots & \ddots & 0 \\ 0 & \dots & 0 & c_0 & c_1 & \dots & c_{L-1} \end{bmatrix}$$

Using eigenvalue decomposition (EVD), we get

$$\mathbf{C_y} - \sigma_x^2 \mathbf{HH^H}$$
$$= \begin{pmatrix} \mathbf{u_s} & \mathbf{U_v} \end{pmatrix} \begin{pmatrix} \lambda + \sigma_v^2 & 0 & \dots & 0 \\ 0 & \sigma_v^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \sigma_v^2 \end{pmatrix} \begin{pmatrix} \mathbf{u_s^H} \\ \mathbf{U_v^H} \end{pmatrix}$$

where $\lambda = P\alpha^2$ and $\mathbf{u_s} = \mathbf{s}$.

Based on this exposition, a direct SB method may be formulated as follows. The autocorrelation matrix $\mathbf{C_y}$ is estimated by time averaging

$$\hat{\mathbf{C}}_\mathbf{y} = \frac{1}{L_1} \sum_{k=0}^{L_1-1} \mathbf{y_k y_k}^H$$

Channel knowledge is assumed at the receiver, and the term $\sigma_x^2 \mathbf{HH^H}$ is calculated and subtracted from $\hat{\mathbf{C}}_\mathbf{y}$. Performing EVD on the resulting matrix, the largest eigenvalue is denoted ρ_0 and the others are $\{\rho_1, \rho_2, \dots, \rho_{P-1}\}$. The AWGN variance is estimated as

$$\hat{\sigma}_v^2 = \frac{1}{P-1} \sum_{i=1}^{P-1} \rho_i$$

Noting that the largest eigenvalue is

$$\rho_0 = \lambda + \sigma_v^2$$

allows us to estimate the NBI amplitude as

$$\hat{\alpha} = \sqrt{\frac{\rho_0 - \hat{\sigma}_v^2}{P}}$$

The eigenvector corresponding to the largest eigenvalue is denoted

$$\hat{\mathbf{s}} = \begin{bmatrix} \hat{s}_0 & \hat{s}_1 & \dots & \hat{s}_{P-1} \end{bmatrix}^T$$

Since

$$s_k = \tfrac{1}{\sqrt{P}} e^{j\omega k} \quad \dots \quad k = 0, 1, \dots, P-1$$

we have

$$\tfrac{s_{k+1}}{s_k} = e^{j\omega} \quad \dots \quad k = 0, 1, \dots, P-2$$

and the frequency ω can be estimated as follows

$$\hat{\omega} = \frac{1}{P-1} \sum_{k=0}^{P-2} Im\left\{ln\left(\frac{\hat{s}_{k+1}}{\hat{s}_k}\right)\right\}$$

Finally, the phase ϕ is estimated as

$$\hat{\phi} = \frac{1}{N_1} \sum_{k=0}^{N_1-1} Im\left\{ln\left(\frac{r_k \cdot e^{j\hat{\omega}k}}{\hat{\alpha}}\right)\right\}$$

Having estimated the parameters $\left(\hat{\alpha}, \hat{\omega}, \hat{\phi}\right)$, the estimate of the NBI $\hat{i}_k$ is subtracted from the signal y_k and OFDM signal reception proceeds as in figure 3.

Figure 4 shows the BER of an OFDM system employing such an NBI cancellation scheme. The data symbols are QPSK, and $\mathbf{c}$ is chosen from the ETSI Hiperlan/2 Channel B standard [8]. The cyclic prefix length is $N_g = L$. The SNR is fixed at 20dB and the SIR is varied between -25dB and 25dB. Two reference BER curves are also shown, that of an

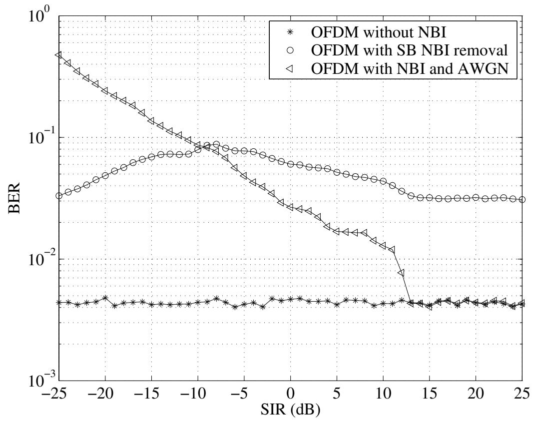

Fig. 4. OFDM system with NBI cancellation by direct application of SB methods to the received signal

OFDM system without NBI, and an NBI corrupted OFDM system without NBI cancellation. It can be seen that estimation of the NBI by direct application of SB methods to the received signal performs quite poorly. In fact for values of $SIR > -10$dB, this NBI cancellation scheme performs worse than the system with no NBI cancellation. This is due to the fact that at high SNR, the noise variance is small, making it difficult to estimate accurately. It can be seen from the relevant equations that inaccurate estimation of σ_v^2 leads to inaccurate estimation of the NBI amplitude $\hat{\alpha}$ and, in particular, the phase $\hat{\phi}$.

IV. Narrowband Interference Cancellation by Cyclic Prefix Aided Subspace Based Methods

We now propose a novel cyclic prefix aided (CPA) SB scheme that facilitates more accurate estimation of the NBI. By subtracting the samples of the cyclic prefix that are unaffected by interblock interference (IBI) from their corresponding samples in the OFDM symbol, the information bearing component is removed from the received signal. Thg noise variance in the resulting signal is doubled, making it easier to estimate. Applying SB methods to the resulting signal results in a dramatic improvement in the accuracy of the NBI estimation.

Employing a cyclic prefix of length N_g, the OFDM transmit signal $\mathbf{x}$ is structured such that

$$x_k = x_{k+N} \quad \dots \quad k = 0, \dots, N_g - 1$$

Due to the IBI caused by the channel impulse response we have

$$z_k \neq z_{k+N} \quad \dots \quad k = 0, \dots, L - 2$$

However, assuming the cyclic prefix is longer than the channel memory, i.e. $(N_g > L - 1)$, we have

$$z_k = z_{k+N} \quad \dots \quad k = L - 1, L, \dots, N_g - 1$$

We denote by $N_2 = N_g - L + 1$ the number of samples used in the NBI estimation process. For each $k = L-1, L, \dots, N_g-1$ we define the sequence q_k as follows

$$\begin{aligned} q_k &\triangleq y_{k+N} - y_k \\ &= \alpha e^{j\phi}(e^{j\omega N} - 1)e^{j\omega k} + w_k \end{aligned}$$

where w_k is AWGN of zero mean and variance $\sigma_w^2 = 2\sigma_v^2$. Defining $L_2 = N_2 - P + 1$ vectors $\mathbf{q_k}$, each of size $(P \times 1)$, as follows

$$\begin{aligned} \mathbf{q_k} &= \left[\begin{array}{cccc} q_k & q_{k+1} & \cdots & q_{k+P-1} \end{array} \right]^T \\ &= \alpha e^{j\phi_k}(e^{j\omega N} - 1)\sqrt{P}\mathbf{s} + \mathbf{w_k} \end{aligned}$$

we may now proceed in a manner similar to the development of section III.

The $(P \times P)$ autocorrelation matrix of the vector $\mathbf{q}$ is by definition

$$\mathbf{C_q} \triangleq E\left\{\mathbf{q_k q_k^H}\right\}$$

and can be written as the sum of a rank one matrix and the identity matrix $\mathbf{I}$ scaled by σ_w^2 as follows

$$\mathbf{C_q} = \alpha^2 P \left| e^{j\omega N} - 1 \right|^2 \mathbf{ss^H} + \sigma_w^2 \mathbf{I}$$

Using EVD, this can be written

$$\mathbf{C_q} = \left(\begin{array}{cc} \mathbf{u_s} & \mathbf{U_w} \end{array} \right) \left(\begin{array}{cccc} \lambda + \sigma_w^2 & 0 & \dots & 0 \\ 0 & \sigma_w^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \sigma_w^2 \end{array} \right) \left(\begin{array}{c} \mathbf{u_s^H} \\ \mathbf{U_w^H} \end{array} \right)$$

where $\lambda = \alpha^2 P \left| e^{j\omega N} - 1 \right|^2$ and $\mathbf{u_s} = \mathbf{s}$.

We generate an estimate of $\mathbf{C_q}$ by time averaging

$$\hat{\mathbf{C}}_\mathbf{q} = \frac{1}{L_2} \sum_{k=L-1}^{N_g-1} \mathbf{q_k q_k^H}$$

Performing the EVD of $\mathbf{C_q}$, the largest eigenvalue is denoted ρ_0 and the others are $\{\rho_1, \rho_2, \dots, \rho_{P-1}\}$ The noise variance σ_w^2 is estimated as

$$\hat{\sigma}_w^2 = \frac{1}{P-1} \sum_{i=1}^{P-1} \rho_i$$

The eigenvector corresponding to the largest eigenvalue is denoted

$$\hat{\mathbf{s}} = \left[\begin{array}{cccc} \hat{s}_0 & \hat{s}_1 & \dots & \hat{s}_{P-1} \end{array} \right]^T$$

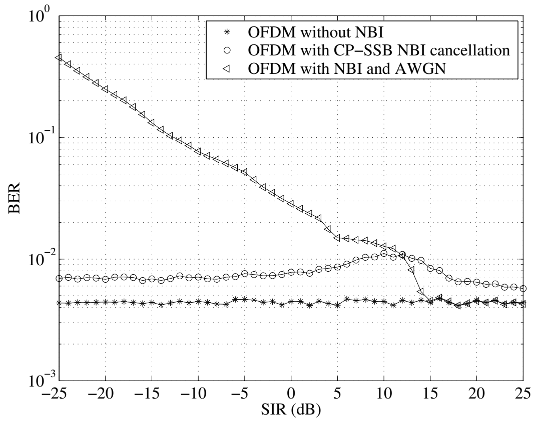

Fig. 5. OFDM system with cyclic prefix aided NBI cancellation using of SB methods

Since

$$s_k = \frac{1}{\sqrt{P}} e^{j\omega k} \quad \dots \quad k = 0, 1, \dots, P-1$$

we have

$$\frac{s_{k+1}}{s_k} = e^{j\omega} \quad \dots \quad k = 0, 1, \dots, P-2$$

and thus the frequency ω can be estimated as follows

$$\hat{\omega} = \frac{1}{P-1} \sum_{k=0}^{P-2} Im \left\{ ln \left(\frac{\hat{s}_{k+1}}{\hat{s}_k} \right) \right\}$$

The largest eigenvalue is

$$\rho_0 = \lambda + \sigma_w^2$$

which allows the estimation of the NBI amplitude as

$$\hat{\alpha} = \sqrt{\frac{\rho_0 - \hat{\sigma}_w^2}{P \left| e^{j\hat{\omega} N} - 1 \right|^2}}$$

Finally the phase is estimated as

$$\hat{\phi} = \frac{1}{N_2} \sum_{k=0}^{N_2-1} Im \left\{ ln \left(\frac{q_k \cdot e^{j\hat{\omega} k}}{\hat{\alpha} \left(e^{j\omega N} - 1 \right)} \right) \right\}$$

Figure 5 shows the results of applying this CPA algorithm to an OFDM NBI corrupted system. Once again, the data symbols are QPSK and the channel $\mathbf{c}$ is chosen from the ETSI Hiperlan/2 Channel B standard [8]. The cyclic prefix length is chosen such that $N_g = N$. The SNR is fixed at 20dB and the SIR is varied between -25dB and 25dB. Two reference BER curves are shown, that of an OFDM system without NBI, and an NBI corrupted OFDM system without NBI cancellation. Comparing the results in figure 5 with that of figure 4, it can be seen that the CPA SB NBI cancellation algorithm vastly outperforms the direct application of SB methods to the received signal. This is due to the fact that the noise variance has been increased, making it easier to estimate accurately. Greater accuracy in the estimation of σ_w^2 results in a more accurate estimation of the NBI amplitude $\hat{\alpha}$ and, in particular, the phase $\hat{\phi}$. Under normal circumstances, the N_2 samples of the received signal unaffected by IBI represent a comfort interval which allows for inaccuracies in the placement of the receiver DFT window. In our CPA algorithm, we take advantage of this comfort interval to perform more accurate estimation of the NBI. It can be seen from figure 5 that the CPA SB algorithm performs close to the case of no NBI for a wide range of SIR.

V. Conclusion

In this paper, we have presented an OFDM system with NBI modelled as a single-tone sinusoid. The deleterious effect of such a NBI was demonstrated by simulation. It was shown how SB methods can be applied to the received signal to estimate the NBI as part of a NBI cancellation scheme. However, at high SNR, the noise variance is difficult to estimate accurately. This leads to a poor estimate of the NBI parameters which causes the NBI estimate to drift in and out of phase with the actual NBI. We then proposed a CPA SB NBI cancellation algorithm. The information bearing component was first removed from the received signal before SB methods were used to estimate the NBI. Simulation results show a dramatic improvement in performance using such a scheme.

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