Pilot Symbol Assisted Turbo Equalization over a Frequency Selective Rayleigh Fading Channel

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Abstract: *A novel pilot-based channel estimation scheme is proposed for frequency selective Rayleigh fading channels which works in conjunction with the existing paradigm of turbo-equalization. The iterative nature of the channel estimation technique provides substantial gain over non-iterative methods, and makes it a suitable choice for iterative equalization and decoding. The use of pilot symbols is demonstrated to aid both the channel estimator and the turbo equalizer independently.*

Keywords: Turbo equalization, channel estimation, pilot symbol assisted modulation, fading channels.

1. Introduction

The "Turbo Principle" applied to the problem of joint equalization and decoding has proven to give remarkable performance over both AWGN and Rayleigh fading channels. However in many cases [1], [2] perfect knowledge of both the time-varying complex channel taps and the additive noise variance was assumed.

Valenti and Woerner [3] proposed a scheme for a flat-fading channel whereby channel estimation could be reiterated at each turbo decoding step, thus improving the overall system performance. In this paper we generalize the iterative channel estimation and decoding technique of [3], making it compatible with Douillard's original scheme [1].

The use of the LMS and RLS algorithms has been explored for iterative estimation and decoding [4], however their tracking capability is quite limited, precluding their use on channels with high fade rates such as are described here.

Also, as will be shown in this paper, the pilot symbols aid both the channel estimator and the MAP equalizer, as opposed to training sequences which aid channel estimation only. This makes pilot symbol assisted modulation a natural choice for this application.

2. System Model

2.1. Transmitter

Figure 1 shows the discrete-time model of the transmitter. N information bits $\{u_k\}, k = 1, 2, \dots N$ are encoded by a rate r parallel concatenated convolutional code (PCCC code) to produce N/r encoded bits $\{c_k\}$. These are then passed to an $R \times S$ block channel interleaver Π_C. The purpose of the channel interleaver is to combat burst errors caused by the fading channel. The resulting bit sequence $\{y_k\}$, $y_k \in \{-1, +1\}$ is parsed into blocks of M bits (M is called the pilot block spacing) and a group of $2L-1$ pilot symbols $\{p_1, p_2, \dots p_{2L-1}\}$ with $p_j \in \{-1, +1\}$ is inserted into the centre of each block (M is assumed even).

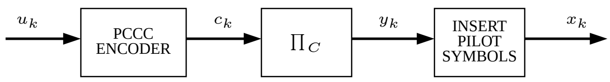

Figure 1: Transmitter Architecture

2.2. Channel

We consider BPSK transmission over an L-tap frequency selective Rayleigh fading channel. The receive signal is given by

$$r_k = \sum_{j=0}^{L-1} h_k^{(j)} x_{k-j} + n_k$$

where each channel tap $h_k^{(j)}$ is a complex Gaussian random variable with zero mean. The different taps are independent and have equal power. The real and imaginary parts of $h_k^{(j)}$ are independent, and both have autocorrelation function given by the Jakes model [5]

$$R_h(i) = \frac{1}{2L} J_0 (2\pi f_d T_s i)$$

where f_d is the maximum Doppler frequency, T_s is the symbol period and J_0 is the zero order Bessel function of the first kind. This normalizes the channel power gain $\sum_{i=0}^{L-1} E\left\{\left|h_k^{(j)}\right|^2\right\}$ to unity. n_k is a complex-valued Gaussian random variable with zero mean and variance σ^2 in each of the real and imaginary directions. The SNR per bit is then $\frac{E_b}{N_0} = \left(\frac{M+2L-1}{M}\right) \cdot \frac{1}{2r\sigma^2}$ with pilot insertion and $\frac{E_b}{N_0} = \frac{1}{2r\sigma^2}$ without.

2.3. Receiver

The proposed receiver architecture is shown in Figure 2. The MAP equalizer and PCCC decoder exchange extrinsic information $\{\lambda^e(x_k)\}$ and $\{\lambda^e(c_k)\}$ on the transmit bits and the PCCC encoded bits respectively, in the manner of Douillard's original scheme [1]. The difference here is that the information corresponding to pilot symbols is removed at the equalizer output as this is of no relevance to the PCCC decoder, and LLRs corresponding to the pilot symbols are inserted in the feedback path. The inserted LLRs are ideally infinite in magnitude as they correspond to *a priori* information on the pilots.

The MAP equalizer requires estimates of the time-varying complex channel taps in order to form its branch metrics. These estimates are provided by a channel estimator described in Section 3.

It is possible to improve the system performance considerably by re-estimating the channel after each (MAP equalizer, PCCC decoder) iteration. This is acheived by including the feedback path shown in dotted lines in Figure 2. The LLRs $\{\lambda^e(c_k)\}$ for the PCCC encoded bits are interleaved and fed to a nonlinear function which transforms the LLRs into bit estimates. The nonlinear function operates according to

$$\hat{y}_k = \begin{cases} \text{sign}\left(\lambda\left(y_k\right)\right) & \text{for hard-decision feedback} \\ \tanh\left(\frac{\lambda(y_k)}{2}\right) & \text{for soft-decision feedback} \end{cases}$$

The original pilot symbols are re-inserted into the bit stream and the resulting estimated transmit stream $\{\hat{x}_k\}$ is used by the channel estimator to produce a refined channel estimate. This improved estimate will aid the MAP equalizer in the next iteration.

3. Channel Estimator

3.1. Known transmit sequence

Here we propose a generalization of Valenti and Woerner's channel estimator [3] to frequency selective channels. Assuming the transmit sequence $\{x_k\}$ is known at the receiver, then the optimum minimum mean-square error (MMSE) estimate of the channel taps is given by [6]

$$\hat{h}_k^{(j)} = \sum_{i=-\left[\frac{K}{2}\right]}^{\left[\frac{K}{2}\right]} w_i^{(j)} r_{k-i} x_{k-i-j} \qquad \text{for } j = 0, 1, \ldots L-1 \tag{1}$$

where $\mathbf{w}^{(j)} = \left(w_{-\left[\frac{K}{2}\right]}^{(j)} \cdots w_{\left[\frac{K}{2}\right]}^{(j)}\right)^T$ is given by the Wiener solution

$$\left(\mathbf{R} + \sigma^2 \mathbf{I}\right) \mathbf{w}^{(j)} = \mathbf{v}$$

Here $\mathbf{R} = (R_{ij}) = (R_h(i-j))$ and

$$\mathbf{v} = \left(R_h\left(-\left[\frac{K}{2}\right]\right) \ldots R_h\left(\left[\frac{K}{2}\right]\right)\right)^T.$$

Thus the channel estimator consists of a bank of Wiener filters, one for each channel tap. Each Wiener filter has the same coefficients and these are real.

If the fade rate is slow enough ($f_d T_s \ll 1$) then the coefficients $w_i^{(j)}$ are all approximately equal, and are

$$w_i^{(j)} = \frac{1}{K + 2L\sigma^2} \approx \frac{1}{K}$$

if $K \gg 2L\sigma^2$. This means that for slow fade rates we can replace the Wiener filters by moving average filters which do not require knowledge of the autocorrelation function of the channel taps or the noise variance, and the noise variance may then be estimated by taking the sample variance of

$$z_k = r_k - \sum_{j=0}^{L-1} \hat{h}_k^{(j)} x_{k-j} \tag{2}$$

3.2. Unknown transmit sequence

Initially the receiver has no knowledge of the transmit symbol sequence other than the pilot symbol values. After the first iteration estimates of the transmit sequence are available via feedback from the PCCC decoder. In both cases it is possible to use a modified version of the scheme described above to perform channel estimation.

3.2.1. Initial channel estimation

We consider a hypothetical transmit sequence $\{\bar{x}_k\}$ made up of a pseudorandom sequence of symbols in $\{-1, +1\}$ (excepting pilots). Consider the first pilot-assisted block for ease of exposition. If the channel taps are reasonably static over the length of this block

$$h_k^{(j)} \approx h^{(j)} \quad k \in \{1, 2, \ldots M + 2L - 1\}$$

The received symbols corresponding to the pilots are

$$\mathbf{p} = \mathbf{A}\mathbf{h} + \mathbf{n}$$

where

$$\mathbf{p} = \left(r_{\frac{M}{2}+L}, r_{\frac{M}{2}+L+1}, \ldots, r_{\frac{M}{2}+2L-1}\right)^T,$$

$$\mathbf{h} = \left(h^{(0)}, h^{(1)}, \ldots, h^{(L-1)}\right)^T,$$

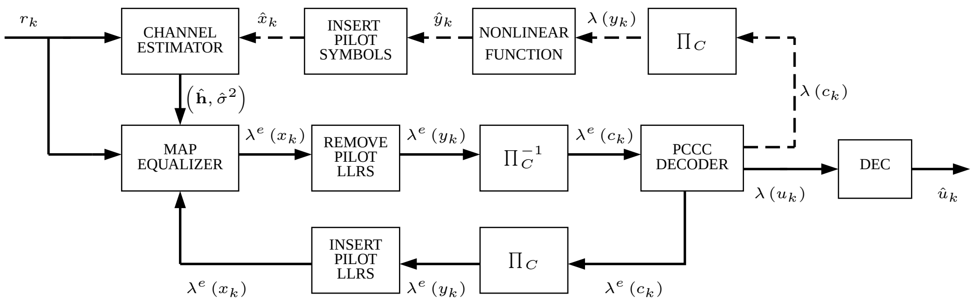

Figure 2: Receiver Architecture

$$\mathbf{n} = \left(n_{\frac{M}{2}+L}, n_{\frac{M}{2}+L+1}, \ldots, n_{\frac{M}{2}+2L-1}\right)^T$$

and

$$\mathbf{A} = \begin{pmatrix} p_L & p_{L-1} & \cdots & p_1 \\ p_{L+1} & p_L & \cdots & p_2 \\ \vdots & \vdots & & \vdots \\ p_{2L-1} & p_{2L-2} & \cdots & p_L \end{pmatrix}$$

Thus, the receive sequence for this block is

$$\bar{\mathbf{r}} = \mathbf{V}\mathbf{A}^{-1}\mathbf{p}$$

where

$$\mathbf{V} = \begin{pmatrix} \bar{x}_1 & 0 & 0 & \cdots & 0 \\ \bar{x}_2 & \bar{x}_1 & 0 & \cdots & 0 \\ \bar{x}_3 & \bar{x}_2 & \bar{x}_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \bar{x}_{M+2L-1} & \bar{x}_{M+2L} & & \cdots & \bar{x}_{M+L} \end{pmatrix}.$$

and the pilots have been chosen to make $\mathbf{A}$ invertible. This receive sequence is computed for each pilot-assisted block. Our initial estimate of the channel taps then proceeds according to

$$\hat{h}_k^{(j)} = \sum_{i=-\left[\frac{K}{2}\right]}^{\left[\frac{K}{2}\right]} w_i^{(j)} \bar{r}_{k-i} \bar{x}_{k-i-j} \qquad \text{for } j = 0, 1, \ldots L-1$$

and then the noise variance is estimated by taking the sample variance of

$$z_k = \bar{r}_k - \sum_{j=0}^{L-1} \hat{h}_k^{(j)} \bar{x}_{k-j}.$$

It is easily seen that this scheme necessitates $2L-1$ pilots in order to work.

3.2.2. Subsequent channel estimation

After the first iteration we can use the estimate of the transmit sequence provided by the PCCC decoder, i.e. we substitute $\hat{x}_k$ in place of x_k in Equations (1) and (2).

4. Simulation Study

We ran simulations for the case of a 3-tap channel ($L = 3$). We used a PCCC code consisting of two rate 1/2 RSC encoders with constraint length 4 and feedback and feedforward generators in octal notation (15_o) and (17_o) respectively. The trellis of the upper encoder was terminated with $m = 3$ tail bits, while the trellis of the lower encoder was left open. A 1250-bit S-random interleaver was used within the PCCC code, with $S = 20$. We punctured half of the parity outputs of the PCCC encoder to give an overall rate of $r = \frac{1}{2}N/(N+m)$. The channel interleaver was a 50×50 block interleaver, and the Wiener filter length was $K = 61$. The pilot sequence used was $\{1, -1, 1, 1, -1\}$ and 12 turbo-equalization iterations were executed using the log-MAP algorithm. In all cases 50 independent frame errors were observed for each point to determine bit error rate.

We tested five different schemes: (a) perfect knowledge of the channel and no transmission of pilot symbols; (b) perfect knowledge of the channel and transmission of pilot symbols; (c) channel estimation with no feedback from PCCC decoder to the channel estimator; (d) hard-decision feedback from decoder to channel estimator; (e) soft-decision feedback from decoder to channel estimator. These five different schemes were compared for the cases of 2 fade rates: $f_d T_s = 0.005$ and $f_d T_s = 0.02$. A pilot block spacing of $M = 20$ was used for the slower fade rate and the moving average approximation to the optimum Wiener filter was used, while for the faster fade rate we chose $M = 10$ and the Wiener solution was required for adequate performance. Of course (a) and (b) are not real "schemes" but used only for the purposes of comparison.

The BER performance of the 5 different schemes is shown in Figure 3 for the slower fade rate $f_d T_s = 0.005$ and in Figure 4 for the faster fade rate $f_d T_s = 0.02$. In the case of perfect channel estimates with no transmission of pilot symbols, the performance is better for the fast fading case. This is because the ability of the interleaver to break up bursts of errors increases with fade rate. The transmission

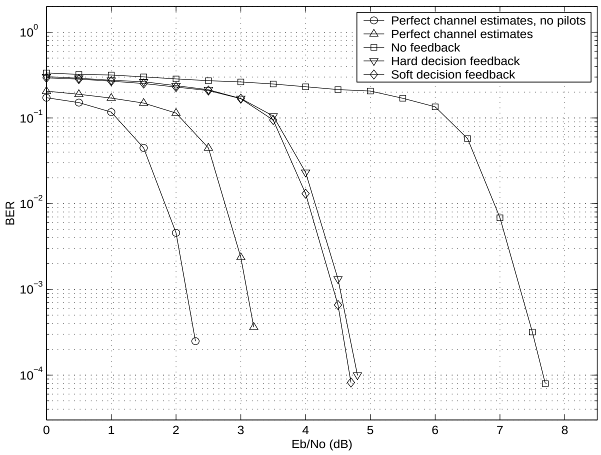

Figure 3: BER performance of the five different schemes with the slower fade rate

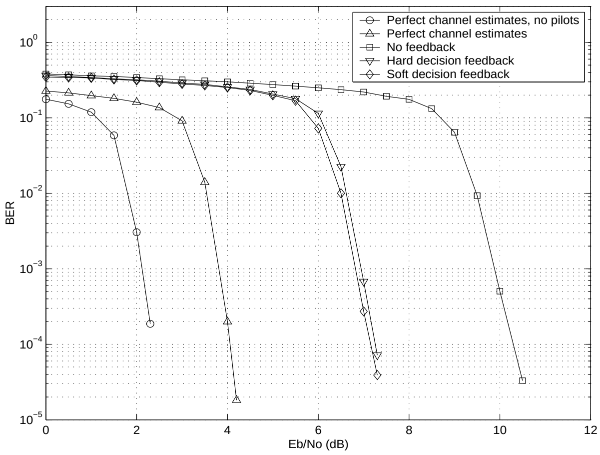

Figure 4: BER performance of the five different schemes with the faster fade rate

of pilot symbols causes a loss in energy efficiency of $10\log_{10}\left(\frac{M+2L-1}{M}\right)$ dB, that is, 0.97 dB for the slower fade rate and 1.76 dB for the faster fade rate. However the performance degradation observed in the simulations was only 0.93 dB for the slower fade rate and 1.69 dB for the faster fade rate at a BER of 10^{-3}. This is because the inserted LLRs corresponding to the known pilot symbols aid the MAP equalizer in determining new extrinsic information.

In the case of channel estimation with no feedback from the PCCC decoder to the estimator, there is a performance degradation of 4.22 dB for the slower fade rate and 6.07 dB for the faster fade rate over the performance for perfect channel estimates, at a BER of 10^{-3}. This degradation can be lessened with the inclusion of feedback from PCCC decoder to estimator. Hard-decision feedback allows us to gain back 2.78 dB for the slower fade rate and 2.94 dB for the faster fade rate at a BER of 10^{-3}. Soft-decision feedback gives an extra 0.1 dB for the slower fade rate and 0.13 dB for the faster fade rate (at a BER of 10^{-3}). For both fade rates the performance improvement when we incorporate feedback into the channel estimator is approximately the same as reported in [3].

5. Conclusion

We have described a scheme for iterative channel estimation, equalization and decoding for pilot symbol assisted BPSK transmission over a frequency selective Rayleigh fading channel. Significant performance improvement is observed for the iterative channel estimation technique over its non-iterative counterpart. Little difference is observed between the performance of hard and soft decision feedback, so we advocate the use of hard decision feedback in practice. The use of pilot symbols is demonstrated to aid independently both the channel estimator and the turbo equalizer.

This work could be extended by the investigation of system performance for higher channel lengths, and the determination of the optimum pilot block spacing M for a particular fade rate and channel length.

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