Adaptive System Identification Based on All Pass/Minimum Phase System Decomposition [1]

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Abstract — **In this paper we present a novel adaptive IIR digital filter composed of a cascade of an all-pass filter and a minimum-phase filter. The coefficients of the filter are updated in such a way as to achieve minimum mean-squared error between the output of the unknown system and the output of the digital filter. Computer simulation verifies that the digital filter coefficients converge to the unknown system coefficients in the case where the system is composed of a cascade of all-pass section and minimum phase section.**

Keywords — **system identification, adaptive filters.**

I Introduction

In the modelling of a system with poles close to the unit circle, it is found that an IIR filter is a much more advantageous model than an FIR filter as it requires fewer taps. Many algorithms have already been proposed for this purpose [1]-[3].

Okello *et al.* [4] have shown that for systems composed of a cascade of an all-pass component and a minimum phase component, a simplified version of the LMS algorithm yields much faster convergence than the conventional IIR algorithms, and also guarantees the convergence of the poles of the filter to those of the unknown system.

The adaptive algorithm presented here identifies systems composed of a cascade of an all-pass component and a minimum phase component, and outperforms Okello's algorithm even in the presence of a coloured input signal.

This paper is organised as follows: Section II describes the structure of the adaptive digital filter, section III introduces the adaptive algorithm, section IV gives the simulation results and section V presents the conclusion.

Notation: For the sequence $\{x_n\}$, we define the vectors $\boldsymbol{x}_k(n) = \begin{bmatrix} x_n & x_{n-1} & \dots & x_{n-k} \end{bmatrix}^T$ and $\text{Bwd}(\boldsymbol{x}_k(n)) = \begin{bmatrix} x_{n-k} & x_{n-k+1} & \dots & x_n \end{bmatrix}^T$. For coefficient vectors, $\boldsymbol{h}_K(n) = \begin{bmatrix} h_0 & h_1 & \dots & h_K \end{bmatrix}^T$ denotes the value of the coefficient vector at time step n. The symbols ^{T} and $E\{\cdot\}$ denote the transpose and the expectation operator, respectively.

II Structure of the Adaptive Digital Filter

The structure of the adaptive digital filter is as shown in Fig. 1. The FIR filter $\{w_j\}$ represents the minimum phase component and the IIR filter $\{c_j\}$ represents the all-pass component. The tap-weight $c_M = 1$ and is not updated by the adaptive algorithm. Note that the section comprised of the feedforward and feedback filters with coefficients $\{c_j\}$ is guaranteed to be all-pass for any $\{c_j\}$ because of the reversal of the coefficients in the feedback part. Note also that during adaptation, the desired output $y(n)$ is provided as input to the feedback section, whereas after the circuit has converged this input is switched to $z(n)$, providing the usual feedback connection. We shall consider that the unknown system is also composed of the cascade of an all-pass component and a minimum phase component, and that it has M poles and N zeros, where $N \geq M$.

III The Adaptive Algorithm

The estimation error at time step n is defined as $e_n = z_n - y_n$, where z_n is the digital filter output and y_n is the desired output. We seek to minimise $J = E\{e_n^2\}$ using an estimate of the gradient of J with respect to the filter coefficients. The output of the first FIR filter (minimum phase component) is

$$q_n = \boldsymbol{w}_N^T(n)\boldsymbol{x}_N(n) \tag{1}$$

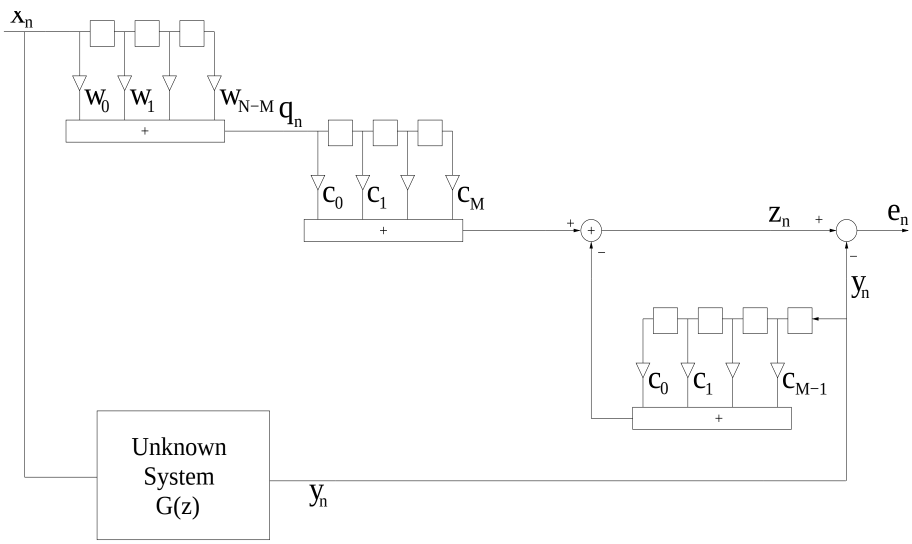

Fig. 1: Structure of the adaptive digital filter.

Thus the output of the adaptive filter is

$$z_n = \boldsymbol{c}_M^T(n)\boldsymbol{q}_M(n) - \boldsymbol{c}_{M-1}^T(n)\mathrm{Bwd}(\boldsymbol{y}_{M-1}(n-1)) \tag{2}$$

and so the estimation error at time step n is

$$\begin{aligned} e_n &= z_n - y_n & (3)\\ &= \boldsymbol{c}_M^T(n)\boldsymbol{q}_M(n) - \\ &\quad \boldsymbol{c}_{M-1}^T(n)\mathrm{Bwd}(\boldsymbol{y}_{M-1}(n-1)) - y(n) & (4)\\ &= \boldsymbol{c}_M^T(n)\boldsymbol{q}_M(n) - \boldsymbol{c}_M^T(n)\mathrm{Bwd}(\boldsymbol{y}_M(n)) & (5)\\ &= \boldsymbol{c}_M^T(n)\{\boldsymbol{q}_M(n) - \mathrm{Bwd}(\boldsymbol{y}_M(n))\} & (6) \end{aligned}$$

Assuming that the adaptation of the coefficients $\{w_j\}$ is slow enough so that the second order statistics of $\{q_n\}$ are approximately time-invariant ("quasi-static approximation"), an estimate of the gradient of J with respect to the $\{c_j\}$ is given by

$$\frac{\delta J}{\delta c_j}(n) \approx 2e_n\{q_{n-j} - y_{n-M+j}\} \quad \text{for } j = 0, 1, \ldots M-1 \tag{7}$$

giving the adaptive update for the $\{c_j\}$ as

$$\begin{aligned} \boldsymbol{c}_{M-1}(n+1) &= \boldsymbol{c}_{M-1}(n) - & (8)\\ \mu e_n\{\boldsymbol{q}_{M-1}(n) &- \mathrm{Bwd}(\boldsymbol{y}_{M-1}(n-1))\} & (9) \end{aligned}$$

For the adaptation of the $\{w_j\}$, we observe that the order of the two cascaded FIR filters can be interchanged to form an equivalent circuit. The output of the first FIR would then be

$$s_n = \boldsymbol{c}_M^T(n)\boldsymbol{x}_M(n) \tag{10}$$

The output of the adaptive filter is in this case

$$z_n = \boldsymbol{w}_N^T(n)\boldsymbol{s}_N(n) - \boldsymbol{c}_{M-1}^T(n)\mathrm{Bwd}(\boldsymbol{y}_{M-1}(n-1)) \tag{11}$$

and so the estimation error at time step n is

$$\begin{aligned} e_n &= z_n - y_n & (12)\\ &= \boldsymbol{w}_N^T(n)\boldsymbol{s}_N(n) - \\ &\quad \boldsymbol{c}_M^T(n)\mathrm{Bwd}(\boldsymbol{y}_M(n)) & (13) \end{aligned}$$

Similarly to the previous case, we assume that the adaptation of the coefficients $\{c_j\}$ is slow enough so that the second order statistics of $\{s_n\}$ are approximately time-invariant. In this case, the MSE gradient estimate is given by

$$\frac{\delta J}{\delta w_j}(n) \approx 2e_n s_{n-j} \qquad \text{for } j = 0, 1, \ldots N \tag{14}$$

giving the adaptive update for the $\{w_j\}$ as

$$\boldsymbol{w}_N(n+1) = \boldsymbol{w}_N(n) - \mu e_n \boldsymbol{s}_N(n) \tag{15}$$

IV Simulation Results

In this section we present the results of computer simulations of the adaptive system estimator. The measurement of performance is the echo return loss enhancement (ERLE) defined as

$$ERLE(n) = 10log_{10}\frac{E\{y_n^2\}}{E\{e_n^2\}} \qquad \text{(dB)} \tag{16}$$

In each case the transfer function of the unknown system can be expressed as

$$G(z) = H_A(z)H_M(z) \tag{17}$$

where

$$H_A(z) = \sum_{i=0}^{N-M} \gamma_i z^{-i} \tag{18}$$

and

$$H_M(z) = \prod_{i=1}^{M} \left(\frac{\alpha_i + \beta_i z^{-1} + z^{-2}}{1 + \beta_i z^{-1} + \alpha_i z^{-2}} \right) \tag{19}$$

In order to perform a direct comparison with the results of Okello *et al.*, we simulated identification of the same systems. In each case the step-size parameter μ was chosen to produce the fastest convergence for each simulation, and the evolution of the ERLE was ensemble averaged over 100 simulations.

Example System 1. First we considered a system with the following parameters,

$$\gamma_0 = 1,\ \gamma_1 = 0.4,\ \gamma_2 = -0.1,$$
$$\alpha_1 = 0.64,\ \alpha_2 = 0.81,\ \alpha_3 = -0.42,$$
$$\beta_1 = -0.8,\ \beta_2 = 0.9,\ \beta_3 = -0.1,$$

and a stationary white input signal. Fig. 2 shows the evolution of the ERLE for the adaptive filter with 6 poles and 8 zeros ($\mu = 0.07$). Okello's algorithm converges to an ERLE of 120 dB in 2000 iterations, whereas our algorithm reaches the same ERLE in 600 iterations.

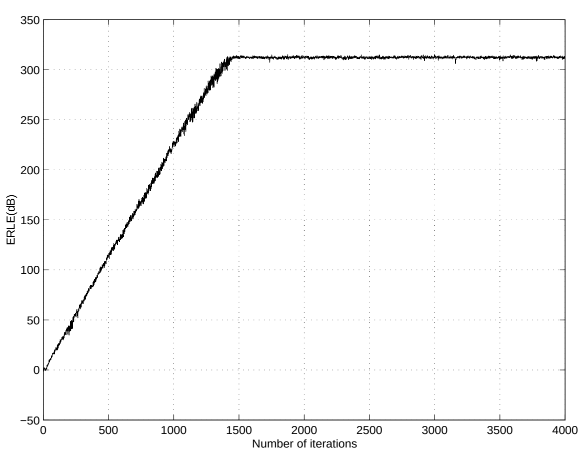

Fig. 2: Convergence characteristic of the adaptive algorithm for example system 1 ($\mu = 0.07$)

Example System 2. Next we considered Okello's second system, with one of the system poles close to the unit circle. This has parameters

$$\gamma_0 = 1,\ \gamma_1 = 0.4,\ \gamma_2 = -0.1,$$
$$\alpha_1 = 0.36,\ \alpha_2 = 0.49,\ \alpha_3 = 0.9025,$$
$$\beta_1 = -0.7713,\ \beta_2 = 1.3155,\ \beta_3 = -0.822.$$

Again the adaptive filter had 6 poles and 8 zeros ($\mu = 0.05$) . Here Okello's algorithm converges to an ERLE of 120 dB in 13000 iterations, whereas our algorithm reaches the same ERLE in 1000 iterations (Fig. 3).

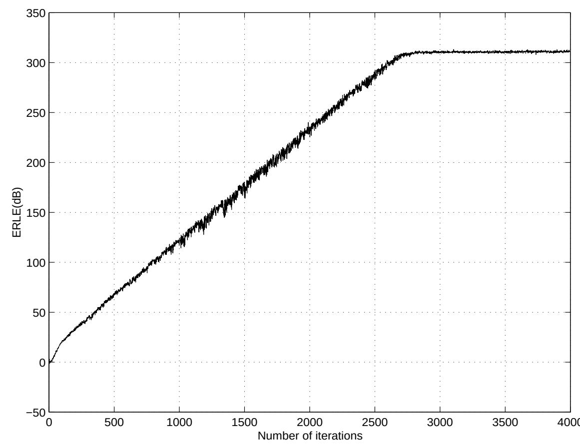

Fig. 3: Convergence characteristic of the adaptive algorithm for example system 2 ($\mu = 0.05$)

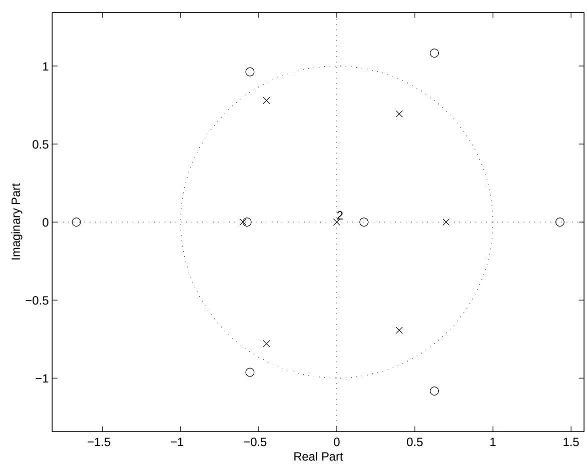

Fig. 4: Pole-zero plot upon convergence of the adaptive system estimator for example system 1, with N = 8 and M = 6.

In the case of example system 1 above, the adaptive filter was given exactly enough parameters to represent the system exactly. The pole-zero plot of the filter upon convergence is given in Fig. 4. We ran the simulation again giving the adaptive filter 9 poles and 13 zeros, and the resulting pole-zero plot is shown in Fig. 5. As can be seen, the system is represented exactly by the filter; redundant pole-zero pairs are produced which cancel each other out.

p	ERLE level	No. of iterations (Okello's algorithm)	No. of iterations (proposed algorithm)
0.3	120 dB	7500	1200
0.6	120 dB	10000	2400
0.8	60 dB	10000	2500

Table 1: Comparison of the number of iterations required to achieve a desired ERLE level (coloured input signal)

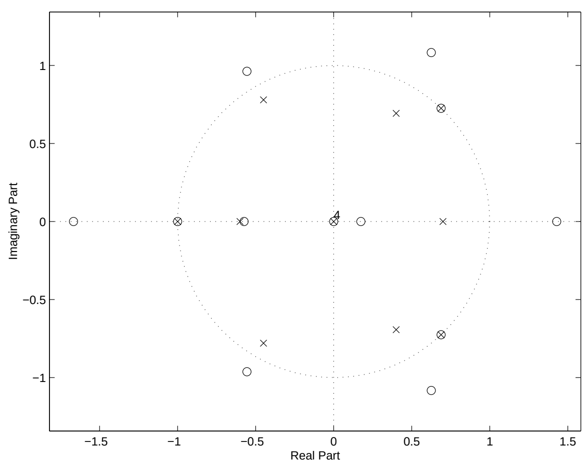

Fig. 5: Pole-zero plot upon convergence of the adaptive system estimator for example system 1, with N = 13 and M = 9.

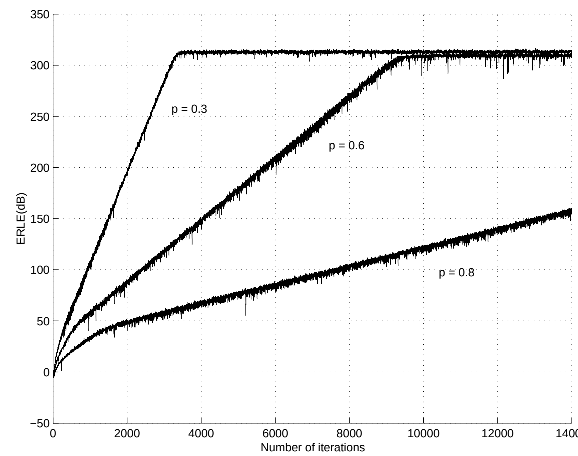

Fig. 6: Convergence characteristic of the adaptive algorithm for the case of a coloured input signal ($\mu =$ 0.04, 0.02 and 0.008 for the cases $p = 0.3$, 0.6 and 0.8 respectively)

Example System 3. In the presence of a coloured input signal $x'(n)$ generated using a stationary white signal $x(n)$, as

$$x'(n) = x(n) + px'(n-1), \qquad |p| < 1 \qquad (20)$$

and using the same unknown system as in Fig. 2, convergence of the ERLE was again found to be much faster than for Okello's algorithm. The convergence plots are shown in Fig. 6 for three different values of p, and a comparison with Okello's results is given in Table 1.

Computational complexity For the exact representation of a sytem with M poles and N zeros ($N \geq M$), Okello's algorithm requires N delay elements whereas the proposed algorithm requires $N + M$ delay elements. Both algorithms involve $N + 1$ LMS-type parameter updates per iteration. Therefore, our proposed algorithm performs markedly better at the expense of a modest increase in memory requirements.

V Conclusion

We have presented a new adaptive filter for the estimation of a system which is composed of a cascade of an all-pass section and a minimum phase section. The gradient estimation algorithm was simple to implement, cost very little computational effort, and converged to the correct solution far more rapidly than Okello's algorithm, even in the presence of a coloured input signal.

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