Conditional quasi maximum-likelihood receiver for clipped OFDM signals

Sebastian Prot[*] Mark Flanagan[*] Conor Heneghan[*]

Abstract **– Orthogonal frequency division multiplexed systems are particularly prone to errors due to clipping, since they typically have high peak-to-average power ratios. In this paper, we propose a new, more efficient, clipping mitigation technique based on quasi maximum-likelihood detection which gives approximately 3 dB gain for the SER of $2\mathrm{x}10^{-3}$ and an SNR of 25 dB.**
Keywords – OFDM, clipping ratio, peak-to-average.

1 INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become increasingly popular as a communication technique for achieving excellent performance with reasonable computational complexity. Multicarrier modulation techniques have been adopted for transmission over Asymmetric Digital Subscriber Line (ADSL) [1], wireless networks based on 802.11a IEEE standard, etc.

The time domain OFDM signal is approximately Gaussian distributed and thus maximizes the system mutual information [6] and hence the overall information throughput of the system, but on the other hand since it is characterized by the high peak-to-average power (PAPR) distribution they cause saturation problems in the digital to analog converters (DAC) and high power amplifiers (HPA) in the transmitter.

In the literature, two approaches have been taken to the PAPR problem:

- relatively complicated techniques for reducing the likelihood of nonlinear distortion, by reducing PAPR in the transmitter [3], and
- less complicated techniques for recovering from the effect of clipping in the receiver[5],[6].

In this paper we will focus on two simple and efficient clipping mitigation techniques called Decision Added Reconstruction (DAR) [5] and Quasi-Maximum Likelihood Detection (Quasi-ML) [6]. We will also propose new scheme that preserve DAR and quasi-ML complexity while improving their SER performance.

2 MATHEMATICAL NOTATION

Unless otherwise stated, we will use the following notation throughout this paper:

- scalars or signals are denoted lower case italic: a;
- time and frequency domain vectors are column vectors and are denoted lower case bold **y** and upper case bold **Y** respectively;
- discrete index sequences are denoted x_k where k is equal to 0, 1, 2,..., N-1;
- all signals are considered discrete-time, Nyquist-sampled.

3 SYSTEM MODEL

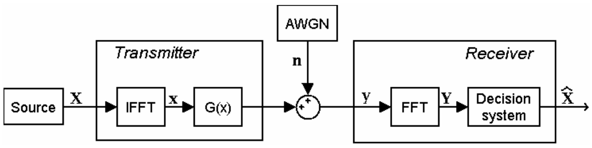

Figure 1: OFDM system model.

We adopt a significantly simplified model of an OFDM system, Figure 1, in which the time domain signal x is generated by means of the inverse discrete Fourier transform (IDFT):

$$x_l = \frac{1}{\sqrt{L}} \sum_{k=0}^{L-1} X_k e^{j2\pi kl/L}, \quad 0 \le l \le L-1 \qquad (1)$$

where x_l and X_k are the l-th time domain (TD) sample and k-th frequency domain (FD) sample respectively, and L represents the overall number of subcarriers. Clipping is performed on the baseband TD signal in the transmitter by a hard limiter [7]:

$$G(x_k) = \begin{cases} x_k & |x_k| < A_{max} \\ A_{max} e^{j\varphi_k} & |x_k| \ge A_{max} \end{cases} \qquad (2)$$

where A_{max} is the threshold amplitude, and ϕ_k denotes the argument of x_k.

The zero mean additive white Gaussian noise (AWGN) channel is considered in this paper, however, all the concepts can be extended to the frequency-selective slow-fading channel which can

[*] Department of Electronic and Electrical Engineering, University College Dublin, Ireland,
e-mail: sebastian.prot@ee.ucd.ie, mark.flanagan@ee.ucd.ie, conor.heneghan@ucd.ie .

be assumed time-invariant over the duration of a communication burst, [5], [6], [9].

According to Bussgang's theorem, [5]-[7], the limiter output, with Gaussian input x, can be modeled as the sum of an attenuated input replica and a distortion term, q uncorrelated with x:

$$G(x_k) = \alpha x_k + q_k \,, \tag{3}$$

where the constant α represents the complex attenuation and phase rotation dependent on the clipping ratio defined below, which approaches 1 for average to high threshold levels [4].

$$\gamma = \frac{A_{max}}{\sqrt{P_x}} \tag{4}$$

Taking the above theorem into consideration, and assuming $\alpha \cong 1$, the received FD signal, after DFT, can be expressed as:

$$Y_k = X_k + Q_k + N_k \,, \tag{5}$$

where Q_k is FD clipping noise and N_k is the FD representation of the AWGN channel noise.

Hard decisions on transmitted OFDM symbols are typically made in the frequency domain as follows:

$$\hat{X}_k = \arg\min_{\bar{X}_k \in S} \left\{ \left| \bar{X}_k - Y_k \right|^2 \right\}, \tag{6}$$

where $\hat{X}_k$ is the estimate of the k-th subsymbol, and $\bar{X}_k$ represents a value from the set of all possible constellation points, *S,* for the k-th subsymbol (notice that each subsymbol can be mapped to different constellations).

4 LOSS OF INFORMATION DUE TO CLIPPING

The mutual information between the transmitted signal x and received signal y is a measure of the maximum achievable data rate per subsymbol transmission, and can be written as [2]:

$$I(x;y) = H(G(x)+n) - H(n) \tag{7}$$

where $H(.)$ denotes entropy – the average information content per symbol.

For the OFDM system with Gaussian input variable x and linear AWGN channel where $G(x) = x$, the mutual information is given by the well known expression:

$$I(x;y) = \tfrac{1}{2}\log_2(1+\tfrac{\sigma_x^2}{\sigma_n^2}) = C[G(x)=x] \tag{8}$$

where σ_x^2 and σ_n^2 denote the variances of the variable x and AWGN respectively, and C denotes channel capacity.

On the other hand if $G(x) \neq x$, then the nonlinearity introduces some loss of mutual information that can be determined by:

$$I(x;y) = H(G(x)+n) - \frac{1}{2}\log_2\left(2\pi e \sigma_n^2\right) \tag{9}$$

where the second quantity is the entropy of the noise variable n [2].

The equation (9) is not however a closed form for general $G(x)$, and direct calculation of mutual information requires numerical integration.

In order to achieve maximum performance the optimum receiver needs to perform ML detection on the received FD signal Y as:

$$\hat{X} = \arg\min_{\bar{X} \in S} \left\{ \left\| (\bar{X} + Q^{(\bar{X})}) - Y \right\|^2 \right\} \tag{10}$$

where the argument is minimized over all possible combinations of $\bar{X}$, and $Q^{(\bar{X})}$ denotes FD clipping distortion.

A suboptimum detection scheme with greatly reduced complexity may be achieved by modeling the FD clipping noise by zero-mean AWGN [4]. In this case the transmission channel reduces to k independent subchannels (see eq. (5)), with additional AWGN noise term Q_k, of variance σ_q^2 [4]-[6]. Taking the above into consideration, the achievable data rate per dimension, per channel becomes:

$$\begin{aligned} I(x;y) &= H(x+n+q) - H(n+q) = \\ &= \tfrac{1}{2}\log_2(1+\tfrac{\sigma_x^2}{\sigma_n^2+\sigma_q^2}). \end{aligned} \tag{11}$$

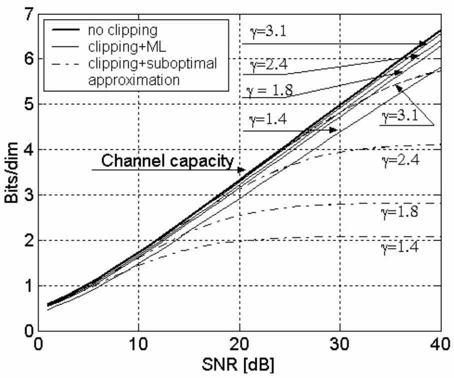

Figure 2: Mutual information vs. SNR for ML receiver, and suboptimum receiver modeling clipping distortion as AWGN. The characteristics are shown for a range of clipping ratios *γ*.

In Figure 2, the mutual information is presented for the three systems:

- linear AWGN channel with Gaussian input,
- a clipped system with maximum-likelihood receiver, and
- the system using the assumption in (11).

The difference between theoretical upper-bound and the actual achievable data rates motivates us to search for the better receiver delivering closer to the optimum SER performance.

6 QUASI-ML SCHEMES

The quasi-ML scheme proposed by Tellado et al. in [6] is a good method for clipping mitigation in the receiver with reduced receiver complexity compared to ML. The quasi-ML scheme serves as a backbone for other methods such as decision added reconstruction [5] (as will be shown later) or quasi-ML scheme with constant attenuation α reconstruction as presented in [10] (see eq. (3)).

In the quasi-ML scheme, the data estimation is performed by making hard decisions in the FD in an iterative manner:

$$\widehat{X}_k^{(i+1)} = \arg\min_{\overline{X}_k \in S} \left\{ \left\| \overline{X}_k - Y_k + Q_k^{(\widehat{X}_k^{(i)})} \right\|^2 \right\} \tag{12}$$

where i denotes the iteration number and $Q^{(\widehat{X}^{(i)})}$ is the distortion estimated during the last iteration.

Intuitively, the estimate of the transmit data, $\widehat{X}$, is composed of the constellation points closest to the sequence $Y - Q^{(\widehat{X}^{(i)})}$. On the other hand, as presented in [5], the clipping mitigation in the DAR method is performed in the time domain by substitution of the received subsymbols y_k by the samples $\widehat{x}_k$ that exceed the threshold amplitude A_{max}:

$$y_k^{(i+1)} = \begin{cases} y_k & |\widehat{x}_k^{(i)}| \le A_{max} \\ \widehat{x}_k^{(i)} & |\widehat{x}_k^{(i)}| > A_{max} \end{cases} \tag{13}$$

where y_k is the signal from channel retrieved from the receiver memory, and $y_k^{(i+1)}$ is used for the data estimation in next iteration (6).

However, both methods can be presented in a convenient form as given for quasi-ML in (12) where the estimate of the clipping distortion is determined in the time domain:

- as a difference between the clipped and actual data estimate, in the quasi-ML scheme:

$$q_k^{(\widehat{x}_k^{(i+1)})} = G(\widehat{x}_k^{(i)}) - \widehat{x}_k^{(i)}\,. \tag{14}$$

- and as a difference between the signal from the channel y_k and the data estimate $\widehat{x}_k$ for $|\widehat{x}_k| > A_{max}$, in the DAR method:

$$q'^{(\widehat{x}_k^{(i+1)})}_k = \begin{cases} y_k - \widehat{x}_k^{(i)} & \text{for } |\widehat{x}_k^{(i)}| > A_{max} \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

The estimate of the clipping noise for the DAR method can be further rewritten as:

$$q'^{(\widehat{x}_k^{(i+1)})}_k = q_k^{(\widehat{x}_k^{(i+1)})} - G(\widehat{x}_k^{(i)}) - y_k = q_k^{(\widehat{x}_k^{(i+1)})} - r_k^{(i)} \tag{16}$$

The term r denotes the additive noise which in most of the cases degrades the clipping distortion estimate q. In consequence, the DAR scheme [5] achieves worse SER reduction performance compared to the quasi-ML introduced by Tellado et al. [6].

In the next section we will propose a novel conditional quasi-ML (CQ-ML) detection scheme which reduces the influence of the overall channel noise on the clipping distortion estimate.

7 CONDITIONAL HARD DETECTION

The conditional quasi-ML receiver can be divided into two stages:

- estimation of the clipping distortion, and
- final estimation of the transmitted data sequence.

During the first stage of the CQ-ML algorithm the estimated time domain signal is used for the detection of the clipping distortion (see e.g. (14)). The better the FD data estimate, the better TD signal matching to the transmitted sequence before clipping. Since the FD false decisions degrade the quality of the FD data estimate, by introducing additional noise, it is convenient to apply the conditional data demodulation scheme as given below.

$$\widehat{X}_k^{(i+1)} = \begin{cases} \arg\min_{\overline{X} \in S} \left\{ \left\| \overline{X}_k - Y_k + Q_k^{(\widehat{X}_k^{(i)})} \right\|^2 \right\} & \text{for } |\mathrm{Re}(\overline{X}_k - Y_k + Q_k^{(\widehat{X}_k^{(i)})})| \le \varepsilon \\ & \text{and } |\mathrm{Im}(\overline{X}_k - Y_k + Q_k^{(\widehat{X}_k^{(i)})})| \le \varepsilon \\ Y_k - Q_k^{(\widehat{X}_k^{(i)})} & \text{otherwise} \end{cases} \tag{17}$$

where ε assumes values between 0 and $0.5d$, where d denotes the Euclidean distance between adjacent constellation points.

The upper part of represents hard decision when the reliability is high enough while the lower part is a reservation of symbol decision otherwise. Some false decisions will still degrade the clipping distortion estimate however careful selection of the decision regions, where the data are considered reliable, significantly improves the SER performance of the system.

8 SIMULATIONS

In our system model only 52 out of 64 subcarriers were used for transmission, thus the 12 subcarriers, including DC, were zeroed as recommended in IEEE 802.11a standard. 48 subcarriers carried the actual data while the last 4 were BPSK modulated pilot signals.

In Figure 3, the SER results for a single iteration of CQ-ML scheme is presented together with performance of the DAR and quasi-ML receivers for various clipping ratios and ε minimizing the SER.

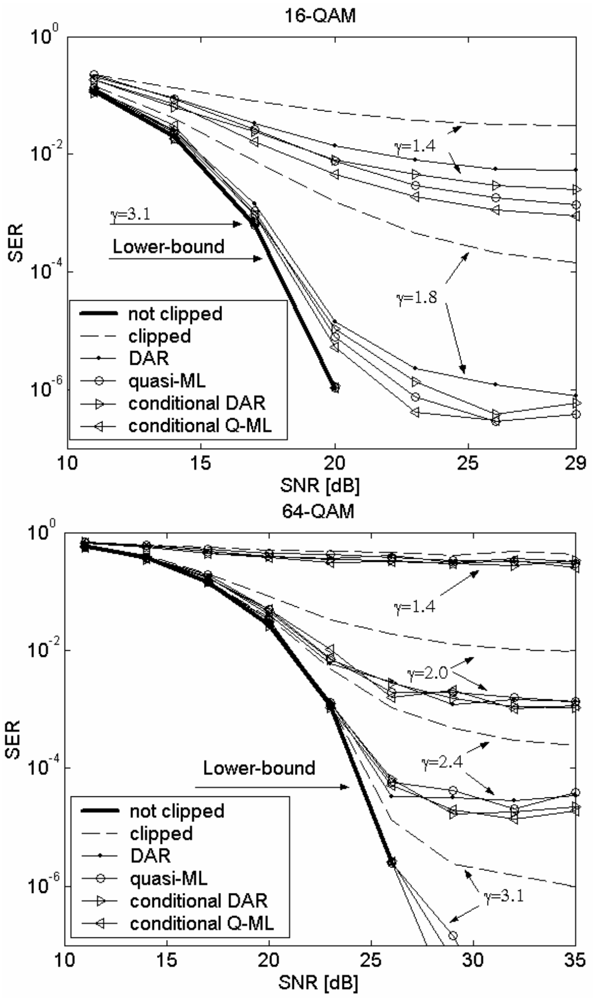

Figure 3: SER performance of OFDM systems with CQ-ML, DAR and quasi-ML receivers for 16-QAM and 64-QAM baseband mapping schemes.

As can be seen in Figure 3, for 16-QAM our CQ-ML scheme introduces:

- 2 to 6 dB gain for SNR of 22 – 29 dB for lowest tested clipping level, and
- 0.5 – 4 dB gain for less severe clipping level i.e. $\gamma = 1.8$ and the same SNR range.

For 64-QAM and both distortion estimation methods, the gain of 1 – 3 dB can be observed for a clipping ratio of 2.4 and SNR greater then 28 dB.

In order to determine the value of ε *which* minimizes the SER, the OFDM transmission was simulated where ε varied between 0 and 0.5 of the Euclidean distance between adjacent constellation points.

A single local minimum of SER was found for all SNRs in the range tested and clipping levels in the range of $0.3d$ - $0.5d$ for 16-QAM, and $0.4d$ - $0.5d$ for 64-QAM baseband mapping scheme.

9 CONCLUSIONS

Conditional hard detection introduces significant SNR gain (equivalent to coding gain), with insignificant increase of system complexity, for the clipping mitigation systems that estimate the clipping distortion from the estimate of the transmit data.

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